

Explaining the anomalous $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays through the hadronic loop effect

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In this work, we carry out the study on $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ ($J = 0, 1, 2$) by considering the hadronic loop mechanism. Our results show that the Belle's preliminary data of the branching ratios for $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ can be well reproduced in our calculation with a common parameter range, which reflects the similarity among these $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays of concern.

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In the past years, the Belle Collaboration has reported some novel phenomena relevant to the hidden bottom decays of $\Upsilon(5S)$. In Ref. [1], Belle indicated that the partial decay widths of $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi^+\pi^-$ are 10^2 times larger than those of $\Upsilon(mS) \rightarrow \Upsilon(nS)\pi^+\pi^-$, where $n, m = 1, 2, 3$ and $m > n$, which is the puzzle in the $\Upsilon(5S)$ hidden-bottom dipion decays. There are two possible explanations for this puzzle. One is that this large decay width can result from the rescattering mechanism, where the hadronic loop composed of the charmed mesons plays an important role [2, 3]. Another possibility is that there is a tetraquark state Y_b near $\Upsilon(5S)$. According to this assumption, Ali *et al.* also studied the $\pi^+\pi^-$ invariant mass spectrum and the $\cos\theta$ distribution of $Y_b \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$. They claimed that the experimental data can be well described under this explanation. However, as indicated in Ref. [4], the result of $Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-$ is not consistent with the corresponding experimental data. That is, in their calculation of $Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-$, they can describe $\pi^+\pi^-$ data. If taking the same parameters to produce the $\cos\theta$ distribution, however, we found that the obtained $\cos\theta$ distribution cannot fit the experimental data. Furthermore, in Ref. [5], the authors also studied $\Upsilon(5S) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ by the rescattering mechanism, where the interference effect was considered. They met the same problem when fitting the experimental data of $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$. Thus, a new puzzle was proposed in Ref. [5]. Later, two charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$ were reported by Belle [6], which also stimulated the authors in Ref. [5] to find the relation between the observed Z_b structures and the solution to this new puzzle. If introducing the intermediate Z_b contributions in $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$, the new puzzle mentioned above can be nicely solved, which also results in the observation of the initial single pion emission mechanism in Ref. [7] to explain why there are two charged Z_b structures near the $B\bar{B}^*$ and $B^*\bar{B}$ thresholds. More theoretical predictions of charged charmonium-like structures around the $D\bar{D}^*$

and $D^*\bar{D}^*$ threshold were, of course, given in Ref. [8].

The studies cited above show that the hadronic loop mechanism, as an important non-perturbative QCD effect, is indeed important to $\Upsilon(5S)$ decays. Before applying the hadronic loop mechanism to study the $\Upsilon(5S)$ decays, this mechanism was extensively applied to study the decays of the higher bottomonium and charmonium in Refs. [9–14] and achieved great successes.

Very recently, Belle announced their observation of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ ($J = 0, 1, 2$), which indicates that the $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays also have large decay widths; i.e., the measured branch ratios of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ are $< 3.4 \times 10^{-3}$, $(1.64 \pm 0.23^{+0.30}_{-0.22}) \times 10^{-3}$, and $(0.57 \pm 0.22 \pm 0.07) \times 10^{-3}$ with $J = 0, 1, 2$, respectively [15, 16]. It should be noticed that even though the tree-level contributions to $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ ($J = 0, 1, 2$) should be strongly suppressed due to the Okubo-Zweig-Iizuka rule, such large decay widths are observed, which again inspires our interest in understanding such quantities. In this work, we propose that the contribution from the hadronic loop should be considered in studying $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$. To give a quantitative answer, we perform the concrete calculation, which is illustrated in the following. This investigation can, of course, provide a good test of the hadronic loop mechanism.

$\Upsilon(5S)$ as a higher bottomonium is above the threshold of a pair of bottom mesons, where $\Upsilon(5S)$ mainly decays into $B^{(*)}\bar{B}^{(*)}$, which means that there exists the strong coupling between $\Upsilon(5S)$ and a bottom meson pair. Thus, the hadronic loop effect can play an important role in the decay of $\Upsilon(5S)$, as just briefly reviewed above. Under the hadronic loop mechanism, these discussed $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ processes occur via the intermediate $B^{(*)}$ meson loops. In Fig. 1, the diagrams describing the $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays are given, where an intermediate bottom meson pair can transit into final states $\chi_{bJ}\omega$ by exchanging a proper bottom meson. Instead of the hadronic description for $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$, we can give a quark level description of the hadronic loop contribution in Fig. 2. Here, a fermion line in red denotes bottom quark while a blue line corresponds to the light quark. $\Upsilon(5S)$ first dissolves into two virtual bottom mesons and then this bottom meson pair can turn into $\chi_{bJ}\omega$ via an exchange of an appropriate bottom meson. The matrix element of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ via hadronic loop

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effect can be depicted as

$$\mathcal{M}(\Upsilon(5S) \rightarrow \chi_{bJ}\omega) = \sum_j \langle \chi_{bJ}\omega | \mathcal{H}_2 | j \rangle \langle j | \mathcal{H}_1 | \Upsilon(5S) \rangle. \quad (1)$$

The corresponding description at the hadron level is listed in Fig. 1.

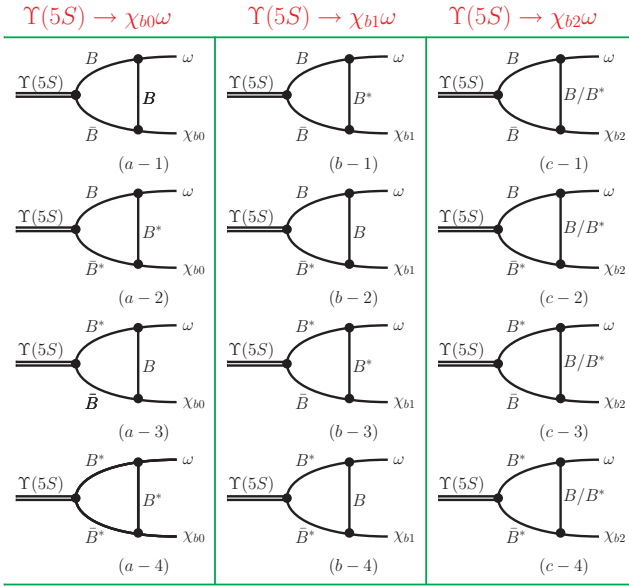


FIG. 1: The necessary diagrams depicting $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays under the hadronic loop effect.

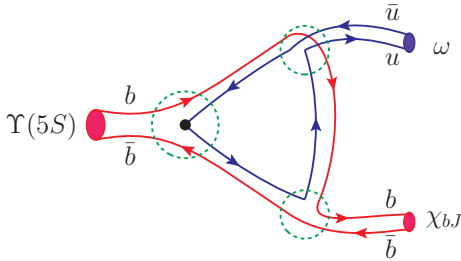


FIG. 2: The quark level diagram depicting $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decay under the hadronic loop effect.

In the heavy quark limit, the wave function of a heavy-light meson is independent of the flavor and spin of the heavy quark; therefore, this wave function can be characterized by the angular momentum of the light degrees of freedom, which is $\vec{s}_\ell = \vec{s}_q + \vec{\ell}$. Each value of $s_\ell = |\vec{s}_\ell|$ corresponds to a degenerate doublet of states with the total angular momentum $J = s_\ell \pm 1/2$. For the bottom meson with $\ell = 0$, the doublet formed by the bottom pseudoscalar and vector meson is represented in [17–20],

$$H^{(Q\bar{Q})} = \frac{1+\not{v}}{2} [\mathcal{B}_\mu^* \gamma^\mu - \mathcal{B} \gamma^5]. \quad (2)$$

For the heavy quarkonium, the degeneracy is expected under the rotations of the two heavy quark spins, although the

heavy quark flavor symmetry does not hold any more. This allows heavy quarkonium with the same angular momentum ℓ to form a multiplet. For the bottomonia with $\ell = 0$, η_b and Υ form a doublet in the form

$$R^{(Q\bar{Q})} = \frac{1+\not{v}}{2} [\Upsilon^\mu \gamma_\mu - \eta_b \gamma_5] \frac{1-\not{v}}{2}. \quad (3)$$

In a similar way, a spin multiplet corresponding to the P -wave bottomonia is,

$$P^{(Q\bar{Q})\mu} = \frac{1+\not{v}}{2} \left[\chi_{b2}^{\mu\alpha} \gamma_\alpha + \frac{1}{\sqrt{2}} \varepsilon^{\mu\alpha\beta\gamma} v_\alpha \gamma_\beta \chi_{b1\gamma} + \frac{1}{\sqrt{3}} (\gamma^\mu - v^\mu) \chi_{b0} + h_b^\mu \gamma_5 \right] \frac{1-\not{v}}{2}. \quad (4)$$

With these multiplets, we can construct the general form of the coupling between heavy quarkonium and heavy meson. The related effective Lagrangians involved in the present work are [19]

$$\begin{aligned} \mathcal{L}_s &= ig \text{Tr} \left[R^{(Q\bar{Q})} \bar{H}^{(Q\bar{Q})} \gamma^\mu \partial_\mu \bar{H}^{(Q\bar{Q})} \right] + H.c., \\ \mathcal{L}_p &= ig_1 \text{Tr} \left[P^{(Q\bar{Q})\mu} \bar{H}^{(Q\bar{Q})} \gamma_\mu \bar{H}^{(Q\bar{Q})} \right] + H.c., \end{aligned} \quad (5)$$

where $H^{(Q\bar{Q})}$ represents the heavy-light meson containing a heavy antiquark \bar{Q} , which can be obtained by applying the charge conjugation operation to $H^{(Q\bar{Q})}$. Expanding the above Lagrangians, we can obtain the following effective couplings:

$$\begin{aligned} \mathcal{L}_{\Upsilon(5S)\mathcal{B}^{(*)}\mathcal{B}^{(*)}} &= -ig_{\Upsilon(5S)\mathcal{B}\mathcal{B}} \Upsilon_\mu (\partial^\mu \mathcal{B} \mathcal{B}^\dagger - \mathcal{B} \partial^\mu \mathcal{B}^\dagger) \\ &\quad + g_{\Upsilon(5S)\mathcal{B}^* \mathcal{B}^*} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \Upsilon_\nu (\mathcal{B}_\alpha^* \partial_\beta \mathcal{B}^\dagger - \mathcal{B} \partial_\beta \mathcal{B}_\alpha^{*\dagger}) \\ &\quad + ig_{\Upsilon\mathcal{B}^* \mathcal{B}^*} \Upsilon^\mu (\mathcal{B}_\nu^* \partial^\nu \mathcal{B}_\mu^{*\dagger} - \partial^\nu \mathcal{B}_\mu^* \mathcal{B}_\nu^{*\dagger} - \mathcal{B}_\nu^* \partial_\mu \mathcal{B}^{*\nu\dagger}), \end{aligned} \quad (6)$$

$$\begin{aligned} \mathcal{L}_{\chi_{bJ}\mathcal{B}^{(*)}\mathcal{B}^{(*)}} &= -g_{\chi_{b0}\mathcal{B}\mathcal{B}} \chi_{b0} \mathcal{B} \mathcal{B}^\dagger - g_{\chi_{b0}\mathcal{B}^* \mathcal{B}^*} \chi_{b0} \mathcal{B}_\mu^* \mathcal{B}^{*\mu\dagger} \\ &\quad + ig_{\chi_{c1}\mathcal{B}\mathcal{B}} \chi_{c1}^\mu (\mathcal{B}_\mu^* \mathcal{B}^\dagger - \mathcal{B} \mathcal{B}_\mu^{*\dagger}) \\ &\quad - g_{\chi_{b2}\mathcal{B}\mathcal{B}} \chi_{b2}^{\mu\nu} \partial_\mu \mathcal{B} \partial_\nu \mathcal{B}^\dagger + g_{\chi_{b2}\mathcal{B}^* \mathcal{B}^*} \chi_{b2}^{\mu\nu} \partial_\mu \mathcal{B}_\nu^* \mathcal{B}_\nu^{*\dagger} \\ &\quad - ig_{\chi_{b2}\mathcal{B}^* \mathcal{B}} \varepsilon_{\mu\nu\alpha\beta} \partial^\alpha \chi_{b2}^{\mu\rho} (\partial_\rho \mathcal{B}^{*\nu} \partial^\beta \mathcal{B}^\dagger - \partial^\beta \mathcal{B} \partial_\rho \mathcal{B}^{*\nu\dagger}), \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{L}_{\mathcal{B}^{(*)}\mathcal{B}^{(*)}\mathcal{V}} &= -ig_{\mathcal{B}\mathcal{B}\mathcal{V}} \mathcal{B}_i^\dagger \partial^\mu \mathcal{B}^j (\mathcal{V}_\mu)_j - 2f_{\mathcal{B}^* \mathcal{B}^* \mathcal{V}} \varepsilon_{\mu\nu\alpha\beta} \\ &\quad \times (\partial^\mu \mathcal{V}^\nu)_j (\mathcal{B}_i^\dagger \partial^\alpha \mathcal{B}^{*\beta j} - \mathcal{B}_i^{*\beta\dagger} \partial^\alpha \mathcal{B}^j) \\ &\quad + ig_{\mathcal{B}^* \mathcal{B}^* \mathcal{V}} \mathcal{B}_i^{*\nu\dagger} \partial^\mu \mathcal{B}_\nu^* (\mathcal{V}_\mu)_j \\ &\quad + 4if_{\mathcal{B}^* \mathcal{B}^* \mathcal{V}} \mathcal{B}_{i\mu}^{*\dagger} (\partial^\mu \mathcal{V}^\nu - \partial^\nu \mathcal{V}^\mu)_j \mathcal{B}_\nu^{*j}, \end{aligned} \quad (8)$$

where \mathcal{V} is the matrix of the vector octet, which is in the form

$$\mathcal{V} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho^0 + \omega) & \rho^+ & K^{*+} \\ \rho^- & \frac{1}{\sqrt{2}}(-\rho^0 + \omega) & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \quad (9)$$

With the above effective Lagrangian, we can write out the amplitudes of hadronic loop contributions to $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ ($J = 0, 1, 2$). For $\Upsilon(5S) \rightarrow \chi_{b0}\omega$, the amplitudes corresponding to Fig. 1 (a-1)-(a-4) are

$$\mathcal{M}_{(a-1)} = \int \frac{d^q}{(2\pi)^4} \left[ig_{\Upsilon BB} \epsilon_{\Upsilon}^{\mu} (ip_{1\mu} - ip_{2\mu}) \right] \left[-ig_{BBV} \epsilon_{\omega}^{\nu} (-ip_{1\nu} - iq_{\nu}) \right] \left[-g_{\chi_{b0} BB} \right] \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{\mathcal{F}^2(\Lambda)}{q^2 - m_B^2}, \quad (10)$$

$$\mathcal{M}_{(a-2)} = \int \frac{d^q}{(2\pi)^4} \left[g_{\Upsilon B^* B} \epsilon_{\rho\mu\alpha\beta} (-ip_0^{\rho}) \epsilon_{\Upsilon}^{\mu} (-ip_2^{\beta} + ip_1^{\beta}) \right] \times \left[-2f_{B^* BV} \epsilon_{\lambda\nu\theta\phi} (ip_3^{\lambda}) \epsilon_{\omega}^{\nu} (ip_1^{\theta} + iq^{\theta}) \right] \left[-g_{\chi_{b0} B^* B^*} \right] \times \frac{1}{p_1^2 - m_B^2} \frac{-g^{\alpha\tau} + p_2^{\alpha} p_2^{\tau} / m_{B^*}^2}{p_2^2 - m_{B^*}^2} \frac{-g_{\tau}^{\phi} + q^{\phi} q_{\tau} / m_{B^*}^2}{q^2 - m_{B^*}^2} \times \mathcal{F}^2(\Lambda) \quad (11)$$

$$\mathcal{M}_{(a-3)} = \int \frac{d^q}{(2\pi)^4} \left[g_{\Upsilon B^* B} \epsilon_{\rho\mu\alpha\beta} (-ip_0^{\rho}) \epsilon_{\Upsilon}^{\mu} (ip_2^{\beta} - ip_1^{\beta}) \right] \times \left[-2f_{B^* BV} \epsilon_{\lambda\nu\theta\phi} (ip_3^{\lambda}) \epsilon_{\omega}^{\nu} (-ip_1^{\theta} - iq^{\theta}) \right] \left[-g_{\chi_{b0} BB} \right] \times \frac{-g^{\alpha\phi} + p_1^{\alpha} p_1^{\phi} / m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2(\Lambda), \quad (12)$$

$$\mathcal{M}_{(a-4)} = \int \frac{d^q}{(2\pi)^4} \left[ig_{\Upsilon B^* B^*} \epsilon_{\Upsilon}^{\mu} (ip_{2\alpha} g_{\mu\beta} - ip_{1\beta} g_{\mu\alpha} - (ip_{2\mu} - ip_{1\mu}) g_{\alpha\beta}) \right] \left[ig_{B^* B^* V} (-ip_{1\nu} - iq_{\nu}) \epsilon_{\omega}^{\nu} g_{\theta\phi} + 4if_{B^* B^* V} \epsilon_{\omega}^{\nu} (ip_{3\phi} g_{\nu\theta} - ip_{3\theta} g_{\nu\phi}) \right] \left[-g_{\chi_{b0} B^* B^*} \right] \times \frac{-g^{\alpha\theta} + p_1^{\alpha} p_1^{\theta} / m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{-g^{\beta\tau} + p_2^{\beta} p_2^{\tau} / m_{B^*}^2}{p_2^2 - m_{B^*}^2} \times \frac{-g_{\tau}^{\phi} + q^{\phi} q_{\tau} / m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}^2(\Lambda), \quad (13)$$

respectively. Similarly, we can write out the amplitudes for $\Upsilon(5S) \rightarrow \chi_{b1}\omega$ corresponding to Fig. 1 (b-1)-(b-4), which are

$$\mathcal{M}_{(b-1)} = \int \frac{d^q}{(2\pi)^4} \left[ig_{\Upsilon BB} \epsilon_{\Upsilon}^{\mu} (ip_{1\mu} - ip_{2\mu}) \right] \left[-2f_{BB^* V} \right] \times \epsilon_{\lambda\nu\alpha\beta} (ip_3^{\lambda}) \epsilon_{\omega}^{\nu} (ip_1^{\alpha} - iq^{\alpha}) \left[ig_{\chi_{b1} B^* B} \epsilon_{\chi_{b1}}^{\theta} \right] \times \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{-g_{\theta}^{\beta} + q^{\beta} q_{\theta} / m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}^2(\Lambda), \quad (14)$$

$$\mathcal{M}_{(b-2)} = \int \frac{d^q}{(2\pi)^4} \left[g_{\Upsilon B^* B} \epsilon_{\rho\mu\alpha\beta} (-ip_0^{\rho}) \epsilon_{\Upsilon}^{\mu} (-ip_2^{\beta} + ip_1^{\beta}) \right] \times \left[-ig_{BBV} (-ip_{1\nu} - iq_{\nu}) \epsilon_{\omega}^{\nu} \right] \left[-ig_{\chi_{b1} B^* B} \epsilon_{\chi_{b1}}^{\theta} \right] \times \frac{1}{p_1^2 - m_B^2} \frac{-g_{\theta}^{\alpha} + p_2^{\alpha} p_{2\theta} / m_{B^*}^2}{p_2^2 - m_{B^*}^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2(\Lambda), \quad (15)$$

$$\mathcal{M}_{(b-3)} = \int \frac{d^q}{(2\pi)^4} \left[g_{\Upsilon B^* B} \epsilon_{\rho\mu\lambda\phi} (-ip_0^{\rho}) \epsilon_{\Upsilon}^{\mu} (ip_2^{\phi} - ip_1^{\phi}) \right] \times \left[ig_{B^* B^* V} (ip_{1\nu} - iq_{\nu}) \epsilon_{\omega}^{\nu} g_{\alpha\beta} + 4if_{B^* B^* V} \right] \times (ip_{3\beta} g_{\nu\alpha} - ip_{3\alpha} g_{\nu\beta}) \epsilon_{\omega}^{\nu} \left[ig_{\chi_{b1} B^* B} \epsilon_{\chi_{b1}}^{\theta} \right] \times \frac{-g^{\lambda\alpha} + p_1^{\lambda} p_1^{\alpha} / m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{1}{p_2^2 - m_B^2} \times \frac{-g_{\theta}^{\beta} + q^{\beta} q_{\theta} / m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}^2(\Lambda), \quad (16)$$

$$\mathcal{M}_{(b-4)} = \int \frac{d^q}{(2\pi)^4} \left[ig_{\Upsilon B^* B^*} \epsilon_{\Upsilon}^{\mu} (ip_{2\alpha} g_{\mu\beta} - ip_{1\beta} g_{\mu\alpha} - (ip_{2\mu} - ip_{1\mu}) g_{\alpha\beta}) \right] \left[-2f_{B^* BV} \epsilon_{\lambda\nu\kappa\phi} (ip_3^{\lambda}) \epsilon_{\omega}^{\nu} (-ip_1^{\kappa} - iq^{\kappa}) \right] \times \left[-ig_{\chi_{b1} B^* B} \epsilon_{\chi_{b1}}^{\theta} \right] \frac{-g^{\alpha\phi} + p_1^{\alpha} p_1^{\phi} / m_{B^*}^2}{p_1^2 - m_{B^*}^2} \times \frac{-g_{\theta}^{\beta} + p_2^{\beta} p_{2\theta} / m_{B^*}^2}{p_2^2 - m_{B^*}^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2(\Lambda). \quad (17)$$

The hadronic loop contribution to $\Upsilon(5S) \rightarrow \chi_{b2}\omega$ is listed in Fig. 1 (c-1)-(c-4). In these diagrams, the exchanged bottom meson can be B meson or B^* meson. The concrete amplitudes are collected in the Appendix.

Considering the isospin symmetry and charge symmetry, we obtain the total amplitude of $\Upsilon(5S) \rightarrow \chi_{b0}\omega$

$$\mathcal{M}_{\Upsilon(5S) \rightarrow \chi_{bJ}\omega}^{\text{Tot}} = 4 \sum_{j=1}^4 \mathcal{M}_{(i-j)}, \quad (18)$$

where $i = a, b$, and c correspond to $\Upsilon(5S) \rightarrow \chi_{b0}\omega$, $\Upsilon(5S) \rightarrow \chi_{b1}\omega$, and $\Upsilon(5S) \rightarrow \chi_{b2}\omega$, respectively. The amplitudes of $\mathcal{M}_{(a-j)}$ and $\mathcal{M}_{(b-j)}$ have been presented in Eqs. (10)-(17). The amplitudes $\mathcal{M}_{(c-i)}$ are defined as $\mathcal{M}_{(c-i)} = \mathcal{M}_{(c-i)}^B + \mathcal{M}_{(c-i)}^{B^*}$. With above amplitudes, the partial decay width reads as

$$\Gamma_{\Upsilon(5S) \rightarrow \chi_{bJ}\omega} = \frac{1}{24\pi} \frac{|\vec{p}_{\omega}|}{m_{\Upsilon(5S)}^2} |\overline{\mathcal{M}_{\Upsilon(5S) \rightarrow \chi_{bJ}\omega}^{\text{Tot}}}|^2, \quad (19)$$

where the overline indicates the sum over the polarization vectors of $\Upsilon(5S)$ and ω . In addition, we define $|\vec{p}_{\omega}| = \lambda^{1/2}(m_{\Upsilon(5S)}^2, m_{\chi_{b0}}^2, m_{\omega}^2)$ with the Källén function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

Adopting the similar approach, we can obtain the amplitudes of $\Upsilon(5S) \rightarrow \chi_{b1}\omega$ and $\Upsilon(5S) \rightarrow \chi_{b2}\omega$, which correspond to Fig. 1 (b-1)-(b-4) and Fig. 1 (c-1)-(c-4), respectively. In the amplitudes, we introduce a form factor in the monopole form to depict the internal structures as well as the offshell effect of the exchanged bottom mesons, where the form factor is taken as $\mathcal{F}(\Lambda) = (m_E^2 - \Lambda^2)/(q^2 - \Lambda^2)$, with m_E the exchanged boson mass. In the heavy quark limit, B and B^* are degenerate and the space wave functions of B and B^* are the same. Thus, in the present work, we parameterize the cutoff Λ as $\Lambda = (m_B + m_{B^*})/2 + \alpha_{\Lambda} \Lambda_{QCD}$ with $\Lambda_{QCD} = 0.22$ GeV.

TABLE I: The coupling constants of $\Upsilon(5S)$ interacting with $B^{(*)}\bar{B}^{(*)}$. Here, we also list the corresponding branching ratios.

Final state	$\mathcal{B}(\%)$	Coupling	Final state	$\mathcal{B}(\%)$	Coupling
$B\bar{B}$	5.5	1.77	$B\bar{B}^*$	13.7	0.14 GeV $^{-1}$
$B^*\bar{B}^*$	38.1	2.25			

Since $\Upsilon(5S)$ is above the threshold of $B^{(*)}\bar{B}^{(*)}$, the coupling constants between $\Upsilon(5S)$ and $B^{(*)}\bar{B}^{(*)}$ can be evaluated by partial decay width of $\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)}$. The partial decay width and the corresponding coupling constants are listed in Table I. If a vector boson multiplet is included, the effective Lagrangian both with pseudoscalars and vector bosons is constructed as in Refs. [17–20]. This Lagrangian includes only one gauge coupling g_1 in the heavy quark limit so that all of the coupling constants are related to this gauge coupling. In the heavy quark limit, the coupling constants of $\chi_{bJ}\mathcal{B}^{(*)}\bar{\mathcal{B}}^{(*)}$ are related to the gauge coupling g_1 by

$$\begin{aligned} g_{\chi_{b0}\mathcal{B}\mathcal{B}} &= 2\sqrt{3}g_1\sqrt{m_{\chi_{b0}}m_{\mathcal{B}}}, & g_{\chi_{b0}\mathcal{B}^*\mathcal{B}^*} &= \frac{2}{\sqrt{3}}g_1\sqrt{m_{\chi_{b0}}m_{\mathcal{B}^*}}, \\ g_{\chi_{b1}\mathcal{B}\mathcal{B}^*} &= 2\sqrt{2}g_1\sqrt{m_{\chi_{b1}}m_{\mathcal{B}}m_{\mathcal{B}^*}}, & g_{\chi_{b2}\mathcal{B}\mathcal{B}} &= 2g_1\frac{\sqrt{m_{\chi_{b0}}}}{m_{\mathcal{B}}}, \\ g_{\chi_{b2}\mathcal{B}\mathcal{B}^*} &= g_1\sqrt{\frac{m_{\chi_{b2}}}{m_{\mathcal{B}}^3m_{\mathcal{B}}}}, & g_{\chi_{b2}\mathcal{B}^*\mathcal{B}^*} &= 4g_1\sqrt{m_{\chi_{b2}}m_{\mathcal{B}^*}}, \end{aligned}$$

where we take the gauge coupling $g_1 = -\sqrt{\frac{m_{\chi_{b0}}}{3}}\frac{1}{f_{\chi_{b0}}}$ and $f_{\chi_{b0}} = 175 \pm 55$ MeV is the decay constant of χ_{b0} [21]. The coupling constants between light vector mesons and bottom mesons are

$$\begin{aligned} g_{\mathcal{B}\mathcal{B}^*\mathcal{V}} &= g_{\mathcal{B}^*\mathcal{B}^*\mathcal{V}} = \frac{\beta g_V}{\sqrt{2}}, \\ f_{\mathcal{B}\mathcal{B}^*\mathcal{V}} &= \frac{f_{\mathcal{B}^*\mathcal{B}^*\mathcal{V}}}{m_{\mathcal{B}^*}} = \frac{\lambda g_V}{\sqrt{2}}, \end{aligned}$$

where the gauge coupling $\beta = 0.9$, $\lambda = 0.56$ GeV $^{-1}$, and $g_V = m_\rho/f_\pi$ with pion decay constant $f_\pi = 132$ MeV [22–25].

With above preparations, we can evaluate the hadronic loop contributions to $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays. The α_Λ is introduced as a free parameter in the cutoff Λ of the form factor. This parameter is usually dependent on particular process and taken to be of the order of unity. In Fig. 3, we present the α_Λ dependence of the branching ratio of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$. The experimental data from the Belle Collaboration [15, 16] are also presented in comparison with our calculated results.

From Fig. 3, we notice that our theoretical estimate can reproduce the experimental data given by the Belle Collaboration [15, 16]. For $\Upsilon(5S) \rightarrow \chi_{b0}\omega$, only the upper limit was given by the experimental measurement, which is $\mathcal{B}(\Upsilon(5S) \rightarrow \chi_{b0}\omega) < 3.4 \times 10^{-3}$, where our result overlaps with the experimental data when taking the range $\alpha_\Lambda < 1.09$. As for the discussed $\Upsilon(5S) \rightarrow \chi_{b1}\omega$ and $\Upsilon(5S) \rightarrow \chi_{b2}\omega$ decays, our calculation can be fitted to the corresponding experimental values when taking $0.41 < \alpha_\Lambda < 0.48$ and $0.43 < \alpha_\Lambda < 0.54$, respectively. Moreover, we need to emphasize that there exists a

common α_Λ range $0.43 < \alpha_\Lambda < 0.48$ for all $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays, which reflects the similarity among these three decays. With this common α_Λ range, we can further restrict the branching ratio of $\Upsilon(5S) \rightarrow \chi_{b0}\omega$, which is $3.00 \times 10^{-4} < \mathcal{B}(\Upsilon(5S) \rightarrow \chi_{b0}\omega) < 4.05 \times 10^{-4}$, where this branching ratio is about 1 order smaller than the corresponding upper limit reported by Belle [15, 16], which can be tested in a future experiment.

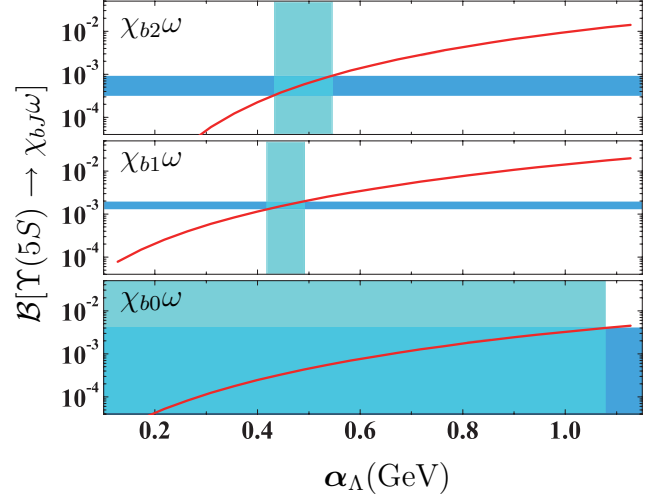


FIG. 3: The branching ratios of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ dependent on the parameter α_Λ . The horizontal bands are the experimental data measured by the Belle Collaboration, while the vertical bands indicate the α_Λ range when our results overlap with the Belle data.

In summary, being stimulated by the recent preliminary results of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ released by Belle [15, 16], we have studied the $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays through the hadronic loop mechanism. In the past years, there were some experimental [1, 6] and theoretical progresses [2, 3, 7, 8] on the $\Upsilon(5S)$ decays, which show that the hadronic loop mechanism can be an important effect on the $\Upsilon(5S)$ decays. The present investigation provides a further test of the hadronic loop effect. Our calculation indicates that the Belle data of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ can be reproduced when the hadronic loop mechanism is considered in $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$. What is more important is that there exists a common α_Λ range for all $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ decays, which is due to the similarity among $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ with $J = 0, 1, 2$. In addition, we further constrain the branching ratio of $\Upsilon(5S) \rightarrow \chi_{b0}\omega$ by the obtained common parameter range, which can be tested in future experiments.

APPENDIX: THE DECAY AMPLITUDES OF $\Upsilon(5S) \rightarrow \chi_{b2}\omega$

We collected the $\Upsilon(5S) \rightarrow \chi_{b2}\omega$ decay amplitudes, i.e.,

$$\mathcal{M}_{(c-1)}^B = \int \frac{d^q}{(2\pi)^4} [ig_{\Upsilon BB} \epsilon_{\Upsilon}^{\mu}(ip_{1\mu} - ip_{2\mu})] [-ig_{BBV}(-ip_{1\nu} - iq_{\nu}) \epsilon_{\omega}^{\nu}] - ig_{\chi_{b2} BB} \epsilon_{\chi_{b2}}^{\alpha\beta}(-ip_{2\alpha})(-iq_{\beta})] \times \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2(\Lambda), \quad (20)$$

$$\mathcal{M}_{(c-1)}^{B^*} = \int \frac{d^q}{(2\pi)^4} [ig_{\Upsilon BB} \epsilon_{\Upsilon}^{\mu}(ip_{1\mu} - ip_{2\mu})] [-2f_{BB^*V} \epsilon_{\lambda\nu\theta\phi} \times (ip_3^{\lambda}) \epsilon_{\omega}^{\nu}(ip_1^{\theta} + iq^{\theta})] - g_{\chi_{b2} B^*B} \epsilon_{\alpha\tau\kappa\zeta}(ip_4^{\kappa}) \epsilon_{\chi_{b2}}^{\alpha\beta} \times (-ip_{2\beta})(-iq^{\zeta})] \frac{1}{p_1^2 - m_{B^*}^2} \frac{1}{p_2^2 - m_{B^*}^2} \times \frac{-g^{\phi\tau} + q^{\phi} q^{\tau}/m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}^2(\Lambda), \quad (21)$$

$$\mathcal{M}_{(c-2)}^B = \int \frac{d^q}{(2\pi)^4} [g_{\Upsilon B^*B} \epsilon_{\rho\mu\tau\kappa}(-ip_0^{\rho}) \epsilon_{\Upsilon}^{\mu}(-ip_2^{\kappa} + ip_1^{\kappa})] \times [-ig_{BBV}(-ip_{1\nu} - iq_{\nu}) \epsilon_{\omega}^{\nu}] - ig_{\chi_{b2} B^*B} \epsilon_{\alpha\theta\lambda\phi}(ip_4^{\lambda}) \epsilon_{\chi_{b2}}^{\alpha\beta} \times (iq_{\beta})(-ip_2^{\phi})] \frac{1}{p_1^2 - m_B^2} \frac{-g^{\tau\theta} + p_2^{\tau} p_2^{\theta}/m_{B^*}^2}{p_2^2 - m_{B^*}^2} \times \frac{1}{q^2 - m_B^2} \mathcal{F}^2(\Lambda), \quad (22)$$

$$\mathcal{M}_{(c-2)}^{B^*} = \int \frac{d^q}{(2\pi)^4} [g_{\Upsilon B^*B} \epsilon_{\rho\mu\tau\kappa}(-ip_0^{\rho}) \epsilon_{\Upsilon}^{\mu}(-ip_2^{\kappa} + ip_1^{\kappa})] \times [-2f_{BB^*V} \epsilon_{\lambda\nu\theta\phi}(ip_3^{\lambda}) \epsilon_{\omega}^{\nu}(ip_1^{\theta} + iq^{\theta})] [g_{\chi_{b2} B^*B^*} \epsilon_{\chi_{b2}}^{\alpha\beta} \times \frac{1}{p_1^2 - m_{B^*}^2} \frac{-g_{\alpha}^{\kappa} + p_2^{\kappa} p_{2\alpha}/m_{B^*}^2}{p_2^2 - m_{B^*}^2} \times \frac{-g_{\beta}^{\phi} + q_{\beta} q^{\phi}/m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}^2(\Lambda), \quad (23)$$

$$\mathcal{M}_{(c-3)}^B = \int \frac{d^q}{(2\pi)^4} [g_{\Upsilon B^*B} \epsilon_{\rho\mu\tau\kappa}(-ip_0^{\rho}) \epsilon_{\Upsilon}^{\mu}(ip_2^{\kappa} - ip_1^{\kappa})] \times [-2f_{BB^*V} \epsilon_{\lambda\nu\theta\phi}(ip_3^{\lambda}) \epsilon_{\omega}^{\nu}(-ip_1^{\theta} - iq^{\theta})] \times [g_{\chi_{b2} BB} \epsilon_{\chi_{b2}}^{\alpha\beta}(-ip_{2\alpha})(-iq_{\beta})] \frac{-g^{\tau\phi} + p_1^{\tau} p_1^{\phi}/m_{B^*}^2}{p_1^2 - m_{B^*}^2} \times \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2(\Lambda), \quad (24)$$

$$\mathcal{M}_{(c-3)}^{B^*} = \int \frac{d^q}{(2\pi)^4} [g_{\Upsilon B^*B} \epsilon_{\rho\mu\tau\kappa}(-ip_0^{\rho}) \epsilon_{\Upsilon}^{\mu}(ip_2^{\kappa} - ip_1^{\kappa})] \times [ig_{B^*B^*V}(-ip_{1\nu} - iq_{\nu}) \epsilon_{\omega}^{\nu} g_{\theta\phi} + 4if_{B^*B^*V}(ip_{3\phi} g_{\nu\theta} - ip_{3\theta} g_{\nu\phi}) \epsilon_{\omega}^{\nu}] - ig_{\chi_{b2} B^*B} \epsilon_{\alpha\zeta\lambda\delta}(ip_4^{\lambda}) \epsilon_{\chi_{b2}}^{\alpha\beta}(iq_{\beta})(-ip_2^{\delta})] \times \frac{-g^{\tau\theta} + p_1^{\tau} p_1^{\theta}/m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{1}{p_2^2 - m_B^2} \times \frac{-g^{\phi\zeta} + q^{\phi} q^{\zeta}/m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}^2(\Lambda), \quad (25)$$

$$\mathcal{M}_{(c-4)}^B = \int \frac{d^q}{(2\pi)^4} [ig_{\Upsilon B^*B^*} \epsilon_{\Upsilon}^{\mu}(ip_{2\delta} g_{\mu\kappa} - ip_{1\kappa} g_{\mu\delta} - (ip_{2\mu} - ip_{1\mu}) g_{\delta\kappa})] [-2f_{B^*BV} \epsilon_{\lambda\nu\gamma\phi}(ip_3^{\lambda}) \epsilon_{\omega}^{\nu}(-ip_1^{\gamma} - iq^{\gamma})] \times [-ig_{\chi_{b2} B^*B} \epsilon_{\alpha\theta\lambda\sigma}(ip_4^{\lambda}) \epsilon_{\chi_{b2}}^{\alpha\beta}(iq_{\beta})(-ip_2^{\sigma})] \times \frac{-g^{\delta\phi} + p_1^{\delta} p_1^{\phi}/m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{-g^{\kappa\theta} + p_2^{\kappa} p_2^{\theta}/m_{B^*}^2}{p_2^2 - m_{B^*}^2} \times \frac{1}{q^2 - m_B^2} \mathcal{F}^2(\Lambda), \quad (26)$$

$$\mathcal{M}_{(c-4)}^{B^*} = \int \frac{d^q}{(2\pi)^4} [ig_{\Upsilon B^*B^*} \epsilon_{\Upsilon}^{\mu}(ip_{2\delta} g_{\mu\kappa} - ip_{1\kappa} g_{\mu\delta} - (ip_{2\mu} - ip_{1\mu}) g_{\delta\kappa})] [ig_{B^*B^*V}(-ip_{1\nu} - iq_{\nu}) \epsilon_{\omega}^{\nu} g_{\theta\phi} + 4if_{B^*B^*V} \times (ip_{3\phi} g_{\nu\theta} - ip_{3\theta} g_{\nu\phi}) \epsilon_{\omega}^{\nu}] [g_{\chi_{b2} B^*B^*} \epsilon_{\chi_{b2}}^{\alpha\beta} \times \frac{-g^{\delta\theta} + p_1^{\delta} p_1^{\theta}/m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{-g_{\alpha}^{\kappa} + p_2^{\kappa} p_{2\alpha}/m_{B^*}^2}{p_2^2 - m_{B^*}^2} \times \frac{-g_{\beta}^{\phi} + q_{\beta} q^{\phi}/m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}^2(\Lambda), \quad (27)$$

which correspond to Fig. 1 (c-1)-(c-4), respectively.

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